WEEKLY TEST SOLUTION
OYJ MATHEMATICS Date : 13 Oct 2019

MATHEMATICS

31. (d) $x = 3 \pm \left(\frac{-2}{\sqrt{17}}\right)(\sqrt{17}),\ y = -6 \pm \left(\frac{3}{\sqrt{17}}\right)(\sqrt{17})$

and $z = 10 \pm \left(\frac{-2}{\sqrt{17}}\right)(\sqrt{17})$.

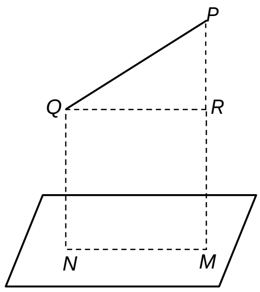
Hence the required co-ordinates are $(1, -3, 8)$ or $(5, -9, 12)$.

32. (a) Centroid $\equiv \left(\frac{\Sigma x}{4}, \frac{\Sigma y}{4}, \frac{\Sigma z}{4}\right) = (1, 2, -1)$

$\Rightarrow a = 1, b = 5, c = -9$; $\therefore \sqrt{a^2 + b^2 + c^2} = \sqrt{107}$.

33. (c) Given plane is $x + y + z - 3 = 0$. From point P and Q draw PM and QN perpendicular on the given plane and $QR \perp MP$.

$|MP| = \frac{0 + 1 + 0 - 3}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{-2}{\sqrt{3}}$, $|NQ| = \frac{-2}{\sqrt{3}}$



$|PQ| = \sqrt{(0-0)^2 + (0-1)^2 + (1-0)^2} = \sqrt{2}$

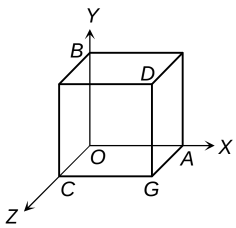
$|RP| = |MP| - |MR| = |MP| - |NQ| = 0$

$\therefore |NM| = |QR| = \sqrt{PQ^2 - RP^2} = \sqrt{(\sqrt{2})^2 - 0} = \sqrt{2}$..

34. (b) Let the cube be of side 'a'

$O(0, 0, 0), D(a, a, a), B(0, a, 0), G(a, 0, a)$

Then equation of OD and BG are $\frac{x}{a} = \frac{y}{a} = \frac{z}{a}$ and $\frac{x}{a} = \frac{y-a}{-a} = \frac{z}{a}$ respectively.



Hence, angle between OD and BG is

$\cos^{-1}\left(\frac{a^2-a^2+a^2}{\sqrt{3a^2}.\sqrt{3a^2}}\right)=\cos^{-1}\left(\frac{1}{3}\right)$.

Note: Students should remember this question as a fact.

35. (a) Line passing through the point (1, 2, –4) is $\frac{x-1}{l}=\frac{y-2}{m}=\frac{z+4}{n}$

Now, according to question, $3l-16m+7n=0$ and $3l+8m-5n=0$

Hence required line is, $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$.

36. (d) We have, $\frac{x-1}{1}=\frac{y+3}{-\lambda}=\frac{z-1}{\lambda}=s$

and $\frac{x-0}{1/2}=\frac{y-1}{1}=\frac{z-2}{-1}=t$

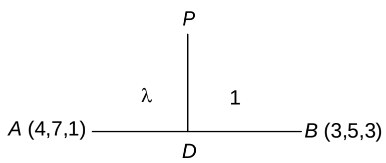
Since, lines are coplanar then

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0\Rightarrow\begin{vmatrix} -1 & 4 & 1 \\ 1 & -\lambda & \lambda \\ 1/2 & 1 & -1 \end{vmatrix}=0$$

On solving, $\lambda=-2$.

37. (b) Let D be the foot of perpendicular drawn from $P(1,0,3)$ on the line AB joining (4, 7, 1) and (3, 5, 3).

If D divides AB in ratio $\lambda:1$ then $D=\left(\frac{3\lambda+4}{\lambda+1},\frac{5\lambda+7}{\lambda+1},\frac{3\lambda+1}{\lambda+1}\right)$ .....(i)



D.r's of PD are $2\lambda+3,5\lambda+7,-2$

D.r's of AB are $-1,-2,2$

$\because\ PD\perp AB$; $\therefore\ -(2\lambda+3)-2(5\lambda+7)-4=0\Rightarrow\lambda=\frac{-7}{4}$

Putting the value of λ in (i), we get the point $D\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$.

38. (b) Any point on $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}=\lambda$ is,

$(2\lambda+1,3\lambda-1,4\lambda+1);\ \lambda\in R$

Any point on $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}=\mu$ is,

$(\mu+3,2\mu+k,\mu);\mu\in R$

The given lines intersect if and only if the system of equations (in λ and μ)

$2\lambda+1=\mu+3$ .....(i)

$3\lambda-1=2\mu+k$ .....(ii)

$4\lambda+1=\mu$ .....(iii)

has a unique solution.

Solving (i) and (iii), we get $\lambda=\frac{-3}{2},\mu=-5$

From (ii), we get $\frac{-9}{2}-1=-10+k\Rightarrow k=\frac{9}{2}$.

39. (b) $\because\ PA^2-PB^2=k$

$\therefore\ [(x-2)^2+(y-3)^2+(z-4)^2]$

$-[(x+2)^2+(y-5)^2+(z+4)^2]=k$

or $-8x+4y-16z-16=k$, which is the equation of a plane.

40. (a) $l+2m+2n=0,\ 3l+3m+2n=0\ , l^2+m^2+n^2=1,$ we get *l, m, n* from these equations and then putting the values in $l(x-1)+m(y+3)\ \ +n(z+2)=0,$ we get the required result.

Trick: Checking conversely,

$2(1)-4(-3)\ +3(-2)-8=0,$

So, it passes through given point.

$1(2)+2(-4)+2(3)=0,$

So, it is perpendicular to $x+2y+2z=5$.

$3(2)+3(-4)+2(3)=0,$

So, it is perpendicular to $3x+3y+2z=8.$

41. (b) The plane by intercept form is $\frac{x}{1}+\frac{y}{1}+\frac{z}{c}=1$.

D.r's of normal are $1,1,\frac{1}{c}$ and of given plane are 1,1, 0. Now, $\cos\frac{\pi}{4}=\frac{1.1+1.1+\frac{1}{c}.0}{\left(\sqrt{\frac{1}{c^2}+2}\right)\sqrt{2}}\Rightarrow$

$$\frac{1}{\sqrt{2}}=\frac{2}{\left(\sqrt{\frac{1}{c^2}+2}\right)\sqrt{2}}$$

$$\Rightarrow\frac{1}{c^2}+2=4\Rightarrow c^2=\frac{1}{2}\Rightarrow c=\frac{1}{\sqrt{2}}$$

$\therefore$ D.r's of required normal are 1, 1, $\sqrt{2}$.

42. (c) Obviously, $4(2)+4(3)-k(4)=0\Rightarrow k=5$.

43. (a) Any point on the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ is $(r+3,\ 2r+4,\ 2r+5)$ which satisfies the plane.

So, $r+3+2r+4+2r+5=17\Rightarrow r=1.$

$\therefore$ The point is (4, 6, 7).

Hence required distance is $\sqrt{1^2+2^2+2^2}=3.$

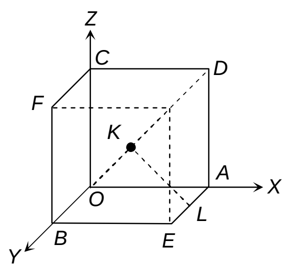
44. (b) We have, $P_1=\left|\frac{3\times2-6\times3+2\times4+11}{\sqrt{3^2+(-6)^2+(2)^2}}\right|=1$

$$P_2=\left|\frac{3\times1-6\times1+2\times4+11}{\sqrt{3^2+(-6)^2+(2)^2}}\right|=\frac{16}{7}$$

So, equation whose roots are P_1 and P_2 is,

$$7P^2-23P+16=0\ .$$

45. (d)



Requaried distance = KL

$$=\sqrt{\left(a-\frac{a}{2}\right)^2+0^2+\left(0-\frac{a}{2}\right)^2}=\frac{a}{\sqrt{2}}.$$